On a Study of Winter Season Wind Structure at 500 mb in the Indian Region for Use in Objective Analysis of the Wind Field

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(Manuscript received 19 October 1972, in revised form 5 March 1973)

ABSTRACT

The statistical macrostructure of the wind field at 500 mb in the Indian region has been studied for the purpose of deriving useful information regarding scan length, scan area and weighting function for Cressman's method of objective analysis.

1. Introduction

Cressman's method, initiated in the Joint Numerical Weather Prediction Unit in 1958, has been adopted as a means of objective analysis of the wind field over the Indian region (Sikka and Ramanathan, 1970). Here we refer to this method as Version I.

The method proceeds by the following steps:

1) A preliminary guess field of the wind components (u and v) to be analyzed is prescribed at the grid points. This may be a prognostic field or a product of a formal extrapolation of analyzed values at previous observational times.

2) Corrections (from current data at station observations) are now introduced into this guess field. The weight factor (W_t) for the data at a station is a function of the distance R between the station and a grid point and is given by

$$W_t=\begin{cases}\dfrac{N^2-R^2}{N^2+R^2}, & \text{for } R<N\\ 0, & \text{for } R\geqslant N\end{cases}$$

where N is the radius of the circular scan.

3) Correction of the preliminary guess field is made from several scans covering the whole field. During each scan the values of the element at all grid points are altered. Each subsequent scan differs from the previous one by a reduction of the range limit N, a technique which corresponds to taking into account features of the observed field which operate on a smaller scale. The scan radii are obtained on empirical considerations only.

4) Since the scans revolve around a grid point, it is assumed that any influence due to changes in the field values is symmetric wlth respect to the grid point.

Sikka and Ramanathan (1970) experimented with three scan distances, 10.0°, 5.0° and 3.5°, and later with four scan distances, 15.0°, 10.0°, 5.0° and 3.5° for both the zonal (u) and meridional components (v) in a circular domain. A 9-point smoothing routine was applied at the end of the last scan with a 3-point smoother on the boundary. The mesh length was taken as 2.5°. The adoption of the above-mentioned scan radii was, however, based on subjective considerations. Although operational testing with this scheme yielded generally satisfactory results, the isotach patterns in a number of cases were not as sharply defined as they should be, especially over regions of wind maxima. Consequently, it was thought that the adoption of scan length, area and weighting function should be based as far as possible on information derived from a study of the macrostructure of the wind field over the region.

Accordingly, in Section 2 the method for computing the structure and correlation functions of any meteorological parameter is described together with the procedure for deriving information about scan length, scan area and weighting function.

In Section 3, numerical computations that assess the nature of the scan area and the scan length of actual cases, are discussed.

In the final section, streamline-isotach analysis on an actual case is shown. This is based on an objective analysis procedure that uses the derived parameters obtained in Section 3, to which we refer as Version II. Comparative results between Versions I and II are also presented.

2. Structure and autocorrelation functions

The scan length, the weighting factor, and the nature of the scan to be used in an objective analysis method may depend on the characteristics of the meteorological field to be analyzed. To this end, we now proceed to construct the structure and correlation functions of the field. The procedure adopted is the same as given by Gandin (1963) and may be briefly stated.

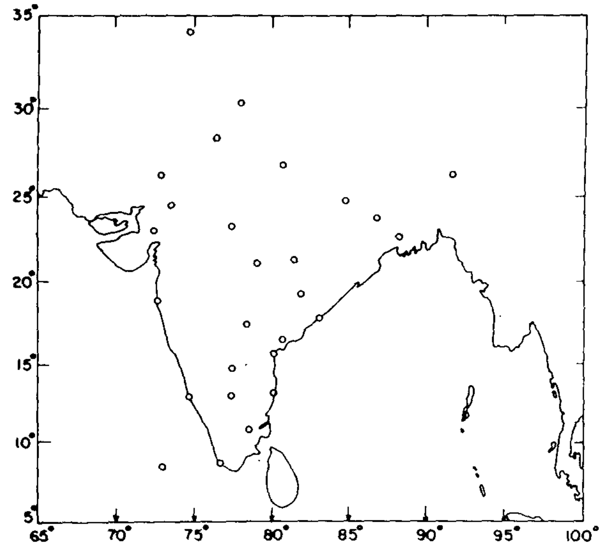

FIG. 1. Observational network used for the computation of the macrostructure of the wind field.

The structure function (SF) of any meteorological field (f) is

$$SF(\mathbf{R}_1,\mathbf{R}_2)=\overline{[f(R_1)-f(R_2)]^2}, \tag{2.1}$$

where $\mathbf{R}_1$ and $\mathbf{R}_2$ are the position vectors of the stations in the region and the overbar denotes averaging. We also define a deviation of the field $f'(\mathbf{R})$ at a station ($\mathbf{R}$) from its normal value $\bar{f}(\mathbf{R})$ as

$$f'(\mathbf{R})=f(\mathbf{R})-\bar{f}(\mathbf{R}). \tag{2.2}$$

The structure function (b_f) for the deviation is defined as

$$b_f(\mathbf{R}_1,\mathbf{R}_2)=\overline{[f'(\mathbf{R}_1)-f'(\mathbf{R}_2)]^2}. \tag{2.3}$$

Similarly the autocorrelation function (CF) for any meteorological field (f) is given by

$$CF(\mathbf{R}_1,\mathbf{R}_2)=f(\mathbf{R}_1)\times f(\mathbf{R}_2), \tag{2.4}$$

and that for the deviation from the mean by

$$m_f(\mathbf{R}_1,\mathbf{R}_2)=f(\mathbf{R}_1)\times f(\mathbf{R}_2). \tag{2.5}$$

From (2.1) and (2.4) it follows that

$$SF(\mathbf{R}_1,\mathbf{R}_2)=CF(\mathbf{R}_1,\mathbf{R}_1)+CF(\mathbf{R}_2,\mathbf{R}_2)-2CF(\mathbf{R}_1,\mathbf{R}_2). \tag{2.6}$$

Similarly from (2.3) and (2.5)

$$b_f(\mathbf{R}_1,\mathbf{R}_2)=m_f(\mathbf{R}_1,\mathbf{R}_1)+m_f(\mathbf{R}_2,\mathbf{R}_2)-2m_f(\mathbf{R}_1,\mathbf{R}_2). \tag{2.7}$$

In addition, if the field is assumed to be isotropic and homogeneous, and if ρ is the distance between the two stations located at points $\mathbf{R}_1$ and $\mathbf{R}_2$, we can write

$$b_f(\rho)=2m_f(0)-2m_f(\rho). \tag{2.8}$$

If the distance $\rho\rightarrow\infty$ (i.e., if the stations are separated by such a large distance that parameters observed at one station have no influence on those at the other), the statistical relationship between the values f' in (2.2) must be damped so that

$$\text{and from (2.8)}\quad \left.\begin{aligned} m_f(\infty)&=0\\ b_f(\infty)&=2m_f(0)\end{aligned}\right\}. \tag{2.9}$$

In the next section, the functional form of the correlation function wlth distance is used to derive the weighting function and the scan length for objective analysis of the zonal and meridional wind components.

3. Numerical computations

A preliminary study was made of the distance variation of the correlation coefficients of the u and v components at 500 mb. For this purpose, three typical stations, Hyderabad (17°27′N, 78°28′E), Nagpur (21°06′N, 79°03′E) and Gwalior (26°14′N, 78°15′E), were selected as central stations. The correlation coefficients of the 500-mb u and v components with respect to 28 wind reporting stations over India (Fig. 1) were obtained for each of these stations, for three winter months, December, January and February, in 1967 and 1968. Since this is usually a fair weather period over the Indian region, pilot balloon observations at different stations were also available on most days. A map of the correlation coefficients was plotted and analyzed for each of the wind components with respect to the three central stations. A typical distrlbution with respect to Nagpur as the central station is shown in Fig. 2. The following features are revealed:

1) The correlation coefficients decrease with distance for both u and v.

2) The correlation coefficient isopleths have an elliptical pattern: the major axis is oriented toward the zonal direction in the case of the u component and in the meridional direction for the v component. From this result, it appears that the circular scanning procedure adopted for objective analysis of the wind field may not be quite suitable, and better results may be expected if the scan area is elliptic. Alaka and Elvander (1972) also computed the autocorrelation coefficients of zonal winds in space over the North American tropics. Their isopleths are also elliptical and thus the fields in the tropics are not strictly isotropic and homogeneous.

Next we proceeded to calculate the two-dimensional structure and correlation functions following the method presented in Section 2. The initial data for these computations were the wind reports at 500 mb for 28 stations (Fig. 1), for three years (1967–69) during the winter period December to February. It is felt that, as far as possible, the initial data are quite homogeneous with respect to randomness of the synoptic situations within the same season. The sample is also sufficiently large and, hence, it can be reasonably expected that the

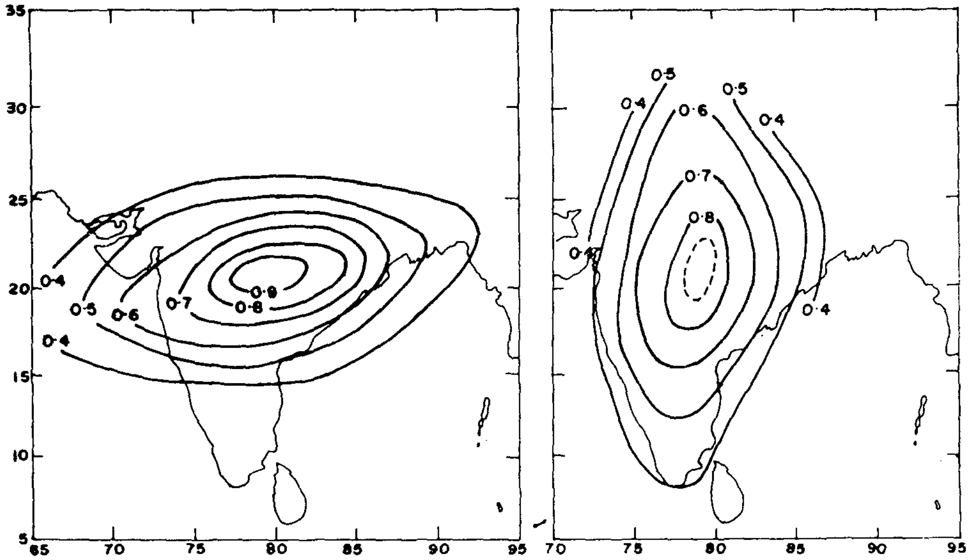

FIG. 2. Distance variation of correlation coefficients of wind components (u, left; v, right) with respect to Nagpur (21°06′N, 79°03′E) as the central station.

introduction of additional data would not significantly change the functions being computed. Computations were made separately for the u and v components, both along the zonal (x) and the meridional (y) directions for all pairs of stations, the maximum distance being about 2000 km. The mean wind for each station was obtained from the data being analyzed. The structure and correlation functions were calculated for a number of distance intervals along the x and y directions, with the total distances being referred to the mid-point of the distance intervals. These results are presented in Table 1.

Regression curves were fitted for the structure and correlation functions of u and v separately with x and y. These are shown in Figs. 3 and 4, using degrees of latitude or longitude as the abscissas. The regression equations (Table 2) for the correlation functions were of the form

$$CF=\alpha_0+\alpha_1 D+\alpha_2 D^2+\alpha_3 D^3, \qquad (3.1)$$

where CF represents u or v as a function of distance D along x or y, and α_0, α_1, α_2, α_3 are constants. Eq. (3.1) is scaled as

$$CF=\alpha_0\left(1+\frac{\alpha_1}{\alpha_0}D+\frac{\alpha_2}{\alpha_0}D^2+\frac{\alpha_3}{\alpha_0}D^3\right). \qquad (3.2)$$

Thus, the derived weight function (WT) to be used for the analysis of the distance variation of the variable is

$$WT=1+\frac{\alpha_1}{\alpha_0}D+\frac{\alpha_2}{\alpha_0}D^2+\frac{\alpha_3}{\alpha_0}D^3. \qquad (3.3)$$

The distance D at which WT vanishes is taken as N, the fundamental scan length, so that the weight function which is 1 at $D=0$ vanishes where $D=N$. This satisfies the twin criteria usually adopted for weighting functions used in objective analysis. The four weight functions

TABLE 1. Structure functions and correlation functions.

Distance x (km)	$b_u(\rho)$	$m_u(\rho)$	$b_v(\rho)$	$m_v(\rho)$	Number of observations N	Distance y (km)	$b_u(\rho)$	$m_u(\rho)$	$b_v(\rho)$	$m_v(\rho)$	N
50	54.7	39.7	34.8	28.1	13463	50	33.4	40.3	40.1	25.5	9795
200	65.5	34.2	38.1	26.4	19664	200	45.0	34.5	47.1	22.0	15379
500	81.3	26.4	49.3	20.9	20284	500	68.9	22.5	57.1	16.9	13798
700	88.4	22.8	59.0	16.1	11482	700	84.5	14.7	61.6	14.7	13014
900	93.9	20.0	69.6	10.8	9622	900	99.1	7.4	64.7	13.1	11832
1100	98.5	17.7	80.4	5.4	8710	1100	111.9	1.0	67.0	12.0	12262
1300	103.1	15.4	90.6	0.3	5160	1300	122.4	−4.2	68.7	11.1	7282
1500	108.5	12.7	99.4	−4.1	5344	1500	129.8	−7.9	70.4	10.3	7538
1700	115.5	9.2	106.1	−7.5	4972	1700	133.5	−9.7	72.3	9.2	6376
1900	124.9	4.5	110.0	−9.4	2030	1900	132.7	−9.3	75.5	7.7	3454
2100	137.6	−1.8	110.2	−9.6	2332	2100	126.9	−6.5	79.7	5.7	1372
2300	154.6	−10.3	105.9	−7.5	974	2300	115.2	−0.6	85.5	2.7	1532
2500	—	—	—	—	—	2500	97.2	8.4	93.5	−1.2	1008

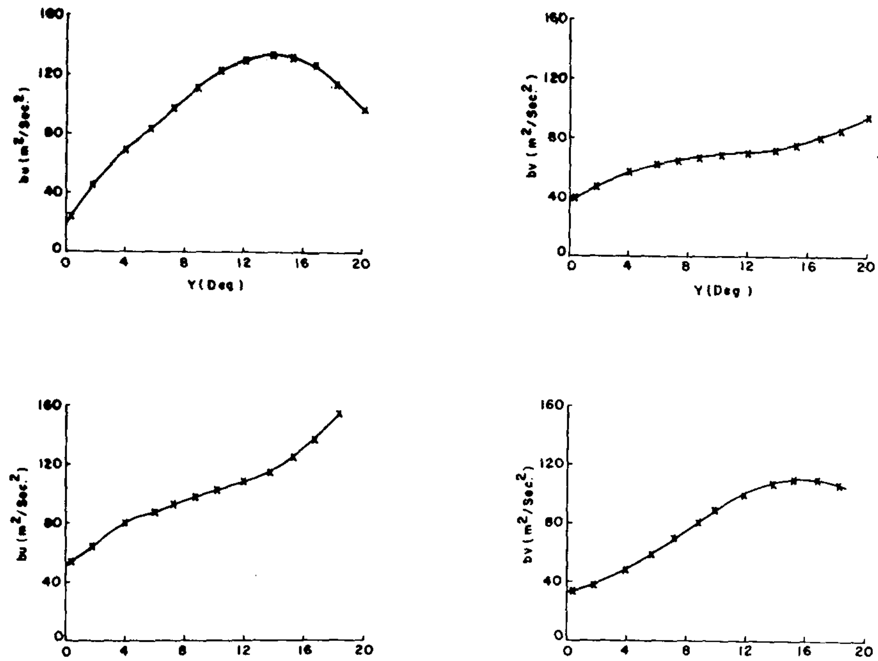

Fig. 3. Structure function variation of u and v along east-west direction x (deg), bottom, and north-south direction y (deg), top.

thus derived for the u and v components along x and y and the distances at which the weight functions become zero are presented in Table 2. Since these are not equal in the x and y directions for both u and v components, separate elliptical scan areas are suggested for the analysis of both u and v. Endlich and Mancuso (1968) also used elliptical weight functions based on wind direction such that greater weight was given for

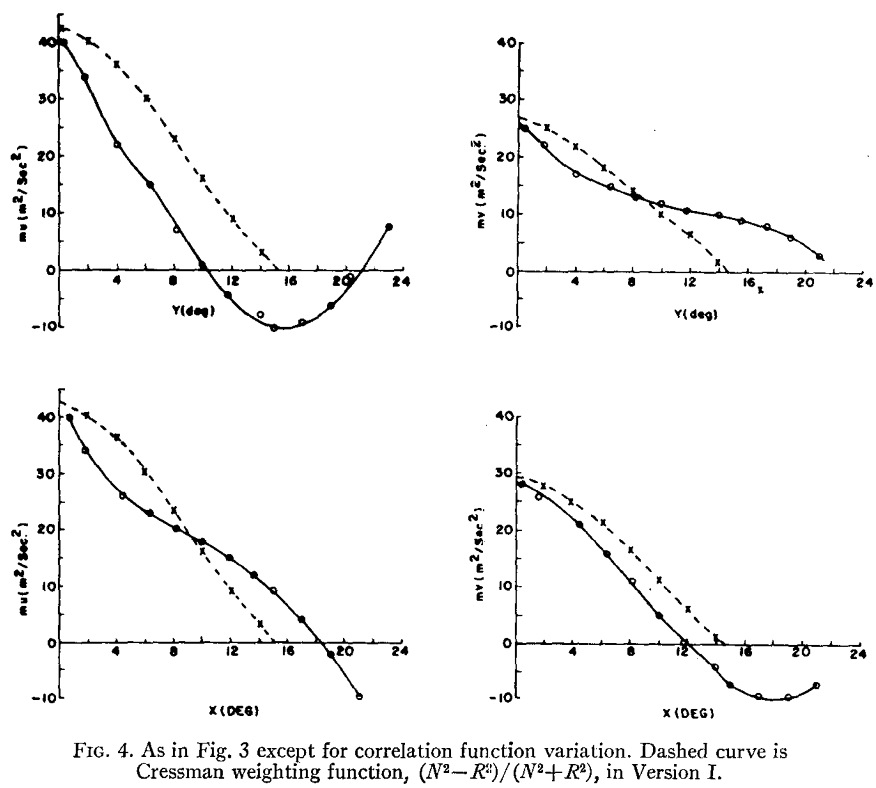

Fig. 4. As in Fig. 3 except for correlation function variation. Dashed curve is Cressman weighting function, $(N^2-R^2)/(N^2+R^2)$, in Version I.

TABLE 2. Regression equations and weight functions.

Parameter (V)	Distance (D)	Regression equation	Weight functions (WT)	Distance (in degrees) at which $WT=0$
u	x	$41.724309-4.715559x$ $+0.34658815x^2-0.01147264x^3$	$1-0.11383555x+0.00836678x^2$ $-0.00027695x^3$	18.4
u	y	$42.131339-4.0437437y$ $-0.09969487y^2+0.00933256y^3$	$1-0.09597872y-0.00236626y^2$ $+0.0002215y^3$	10.6
v	x	$28.406878-0.62787510x$ $-0.27200254x^2+0.01051105x^3$	$1-0.0221092x-0.00957523x^2$ $+0.00037001x^3$	11.8
v	y	$26.77052-3.00733140y$ $+0.21233233y^2-0.00590195y^3$	$1-0.11233742y+0.00793157y^2$ $-0.00022046y^3$	22.4

observations along the flow than for those across the flow.

The structure function vs distance (Fig. 3) for the u component along y and for the v component along x show humps at a distance of about 16° (~1700 km), where they reach saturation values. However, the structure function curves for u with x and v with y do not show any such saturation values within the maximum computational distance (2500 km) taken in this study. The correlation function (Fig. 4) falls more rapidly for u with y and v with x, in comparison to u with x and v with y. This is in keeping with the varia-

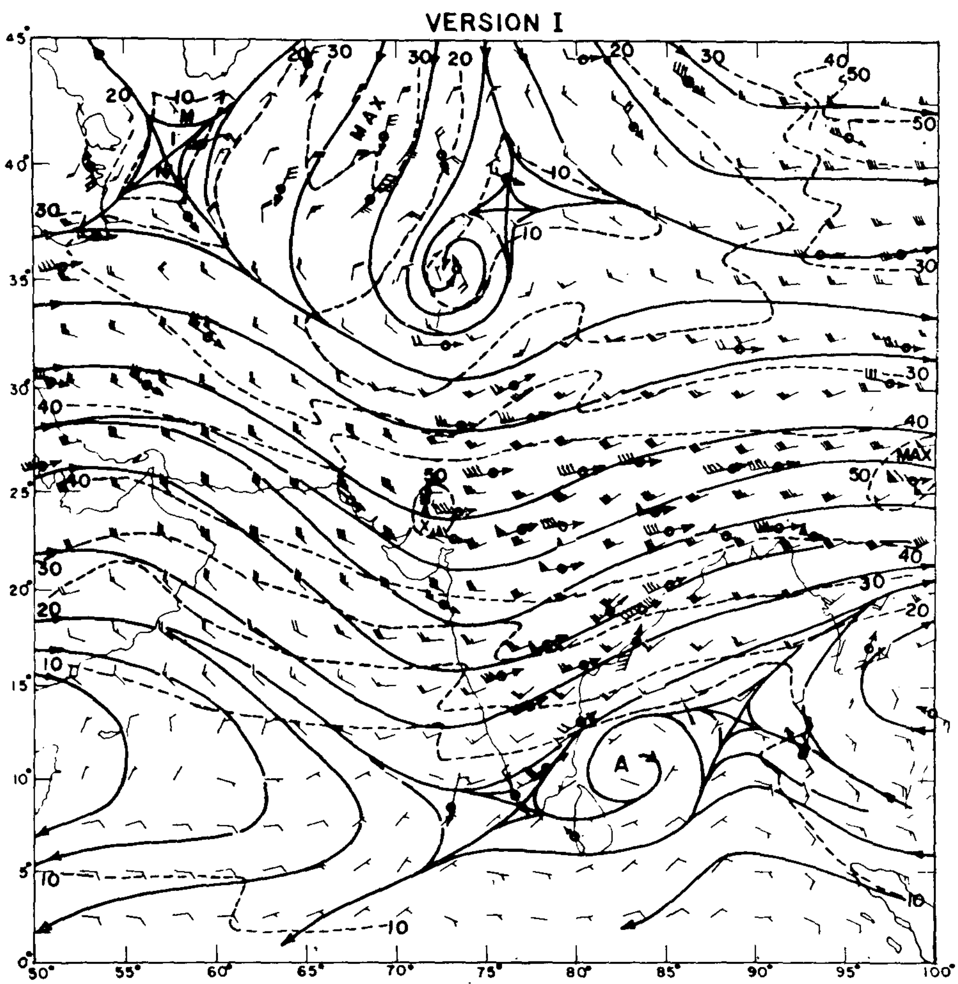

FIG. 5. Streamline-isotach analysis at 500 mb on 25 December 1968 at 0000 GMT based on objective analysis by Version I. Observed wind vectors are shown on the station circles with arrow heads showing the wind direction and barbs, the speed.

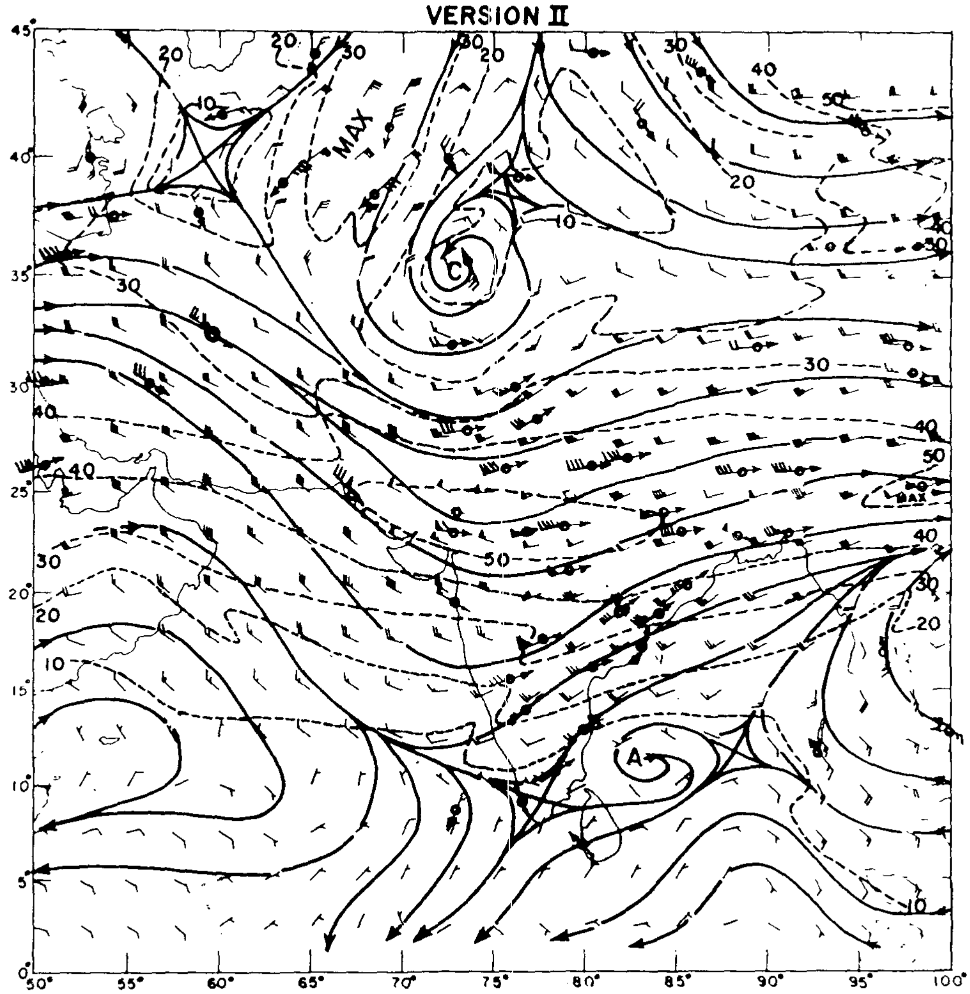

FIG. 6. As in Fig. 5 except for Version II.

tion of structure functions discussed earlier. The correlation function values fall more rapidly with distance for u with y and v with x, than the Cressman's weighting function values used in Version I (Fig. 4). The same pattern is followed in the other two cases up to a distance of about 8° (~900 km) only. Beyond this distance, they fall less rapidly than Cressman's weighting function values, implying that the influences vanish only at very large distances.

4. Comparison of Version I and Version II analyses

In order to find out whether the derived weighting functions and the scan lengths will improve actual analyses, streamline-isotach patterns were made using both versions. These patterns and the corresponding manual analyses were compared with respect to the chief synoptic features and the position of the maximum isotach. In these experiments, the initial guess field, as well as the station reports used for the analyses, were identical, the initial guess field being the previous synoptic hour manual analysis. Version I, which uses the customary Cressman's weighting function $(N^2-R^2)/(N^2+R^2)$, where N is the scan radius and R the distance of the station from the grid point to be corrected, utilized circular scanning with radii of 15°, 10°, 5° and 3.5° for both u and v components. In Version II, we adopted an elliptical scanning procedure, the major axis being along the zonal and the meridional directions for u and v analyses, respectively.

The scan lengths used in the analysis are given below:

Component	Distance	Scan lengths (deg)
u	zonal (along x axis)	20, 10, 5, and 3.5
u	meridional (along y axis)	10, 5, and 3.5
v	zonal (along x axis)	10, 5, and 3.5
v	meridional (along y axis)	20, 10, 5, and 3.5

These scan lengths are based on the correlation function data discussed in Section 3. In this approach, we consider the distance at which the correlation function becomes zero as the most suitable maximum scan length. However, in practical analysis we have slightly departed from this distance in order to bring uniformity to the adoption of scan radii. This minor departure does not seem to introduce any observable differences in the analyses. To accommodate the smaller wavelengths, subsequent smaller radii are multiples or near multiples of the maximum scan length. This is a normal procedure, discussed by the authors (1972).

The analyses have been made for the region between 2.5 to 42.5N and 50 to 100E, using a grid interval of 2.5° latitude/longitude. One such example of analyses made by Versions I and II is shown in Figs. 5 and 6, respectively. The station observations have been also shown on both charts. The case chosen is that of 25 December 1968 at 500 mb. The chief synoptic features are the subtropical ridge lying between 10 and 15N and a large-amplitude trough in the subtropical westerlies running along about 73E having a cyclonic circulation centered near 35N and 73E. The axis of the belt of maximum westerlies runs along about 25N. Comparing the two charts with the manually analyzed chart shows that there is good agreement in the position of the subtropical ridge line, the center of the anticyclonic circulation imbedded in it to the east of Ceylon, and the center of cyclonic circulation near 35N, 73E. However, the elongated position of the 50-kt isotach maximum over central India and the adjoining area is only well defined in Version II, and is in close agreement with the manual analysis. This latter analysis has not been depicted since the purpose has been largely achieved by plotting the station observations in Figs. 5 and 6. Several other cases which were examined (but not reproduced here) also show that contrary to Version I where a tendency of flattening of the maxima was evident, in Version II the positioning of the maximum isotach pattern in the westerlies was much more realistic. These comparisons show that the procedure being suggested here for analyzing winds over tropical and subtropical belts in which the scanning lengths are different in x and y directions for u and v components, is expected to give better results when the scan lengths are derived by statistical analysis of the wind field. The rms errors (kt) between the reported winds and those interpolated from the analysis are as follows:

	u component	v component
Version I	6.5	6.8
Version II	6.2	6.5

The difference in the rms errors between the two versions is not very significant. Similar results were obtained in other cases examined in this study. Although it would be difficult to distinguish between the analyses made by Versions I and II based on rms differences and on the positions of the centers of cyclonic or anticyclonic circulations, the improvement in the acceptability of the maximum isotach pattern in Version II does have extra merit.

5. Conclusion

A study of the statistical properties of the wind field at 500 mb during the winter season has been made to obtain the largest scan length in both the zonal and meridional directions and the weighting function which can then be used in the objective analysis of the wind components. The analyses obtained by using these parameters have been found to orient the maximum isotach field in the wintertime, subtropical westerlies more satisfactorily than the analyses made by using parameters based on empirical considerations. Thus, a study of the structure of the meteorological fields appears to be extremely useful for adopting parameters for their objective analyses. It is realized that the structure of the fields will vary with the seasons, and that this will lead to new sets of parameters for objective analyses. This appears particularly so in the Indian tropics which experience dominant seasonal reversals in the circulation patterns. It is thus proposed to extend this study to the other seasons, *viz.* southwest monsoon (June–September), premonsoon (March–May) and post monsoon (October–November).

Acknowledgments. The authors are extremely thankful to Dr. K. R. Saha for his interest and his constant encouragement during this study.

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